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# Rasch Analysis of Rank-Ordered Data

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Theoretical and practical aspects of several methods for the construction of linear measures from rank-ordered data are presented. The final partial-rankings of 356 professional golfers participating in 47 stroke-play tournaments are used for illustration. The methods include decomposing the rankings into independent paired comparisons without ties, into dependent paired comparisons without ties and into independent paired comparisons with ties. A further method, which is easier to implement, entails modeling each tournament as a partial-credit item in which the rank of each golfer is treated as the observation of a category on a partial-credit rating scale. For the golf data, the partial-credit method yields measures with greater face validity than the paired comparison methods. The methods are implemented with the computer programs *Facets* and *Winsteps*.

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Rank-ordered data have characteristics that align well with the family of Rasch measurement models. Ranks are observations of elements implying qualitatively more, ordered along an implicit or explicit variable. A single set of ranks, called here a "ranking", contains only enough information to order the elements. If there are two or more rankings of the same elements, then there may be enough information to construct interval measures of the distances between the elements. The interval measures support inferences about future performance, and also investigation into the consistency of particular rankings, or of the ranks of an element across rankings.

In this paper, the rankings of golfers in the 47 USPGA-accredited stroke-play golf tournaments of 2004 will be used illustratively. Vijay Singh won the 46th Tournament, the Chrysler Classic, becoming the first-ever golfer to win $10 million in prize money in one year. A total of 680 golfers are listed as participating in those 47 tournaments. Of these, 17 golfers did not post a final score in at least one tournament. In addition, 306 players had a final score in only one tournament. These included many international and veteran players, such as Tom Weiskopf. A further 52 players participated in two tournaments, including Jack Nicklaus and Arnold Palmer. At the other extreme, one player participated in 36 tournaments, Esteban Toledo. Vijay Singh played in 28 tournaments, Tiger Woods in 18. On average, 133 golfers participated in each tournament.

## Paired Comparisons without Ties

The simplest ranking comprises of two elements. This is a paired comparison. In the ESPN World Ranking after the 45th Tournament, the top two players were Vijay Singh (VS) and Ernie Els (EE). These two golfers played in the same tournaments 13 times. In the first 12 of these tournaments, VS was better ranked 6 times, and EE better ranked six times. This would estimate the two players to have the same golfing ability. In the 13th tournament, VS ranked higher than EE. A ratio-scale "odds" comparison of the abilities, $b_{VS}$ and $b_{EE}$, of VS and EE would be $b_{VS} / b_{EE} = 7/6$, or in interval "log-odds" scaling,

$$\log (b_{VS} / b_{EE}) = B_{VS} - B_{EE} = \log(7/6) = 0.15 \quad (1)$$

A general form of this, similar to models proposed by Bradley and Terry (1952) and Luce (1959), is:

$$\log (P_{nm} / P_{mn}) = B_n - B_m \quad (2)$$

where $P_{nm}$ is the probability that element $n$ ranks higher than element $m$.

For estimation purposes this becomes:

$$B_n - B_m = \log (P_{nm} / P_{mn}) \approx \log (T_{nm} / T_{mn}) \quad (3)$$

where $T_{nm}$ is the number of tournaments in which golfer $n$ is ranked above golfer $m$ and *vice-versa*. The standard error of the measure difference $B_n - B_m$ is

$$SE = ((T_{nm} + T_{mn})/(T_{mn*}T_{nm}))^{½}. \quad (4)$$

This model can be implemented directly in the Rasch software program *Facets* (Linacre, 2004a). When this is done, estimates are obtained more speedily and robustly when each data point is entered twice, once as $B_n$ vs. $B_m$ and again as $B_m$ vs. $B_n$, then weighted 0.5.

Implementing this model in other standard Rasch software is straightforward. The usual rectangular dataset format is that items are columns and persons are rows. In this golfing example, each tournament would be an item, and each player paired-comparison a row. The player with the higher ranking is scored "1", and the other player "0". Analysis of this data set will give the expected measure difference of 0.15 logits with software based on conditional maximum likelihood estimation (CMLE), but will yield a measure difference of 0.30 = 2*0.15 for software based on joint maximum likelihood estimation (JMLE). This estimation bias was noted by Anderson (1973) and also its correction: divide measure difference by two.

In the example, the *SE* of $B_{VS} - B_{EE}$ is $((6+7)/(6 * 7))^{½} = 0.56$ logits. Under JMLE, 0.56 is reported as the S.E. of each of $B_{VS}$ and $B_{EE}$, so the joint *SE* of $B_{VS} - B_{EE}$ is overestimated as $0.56 * 2^{½}$. Accordingly, each JMLE element *SE* needs to be divided by $2^{1/2}$. This is done routinely in *Winsteps* (Linacre, 2004b) when *Paired* = *Yes* is specified.

## Rank Orders as Multiple Independent Paired Comparisons

Ernie Els (EE) and Bill Glasson (BG) both played in 16 tournaments, but never in the same tournament. How are they to be compared? They can be compared through their play against other players. Each ranking can be decomposed into a set of apparently independent paired comparisons, and these can be become the basis for constructing a measurement framework within which all the golfers can be measured. A convenient group of golfers are the 30 who took part in the first tournament of 2004, the Mercedes Championship. EE was one of these, but BG was not, so he is added to the analysis. From the participation of these 31 golfers in the 47 tournaments, 5,220 paired comparisons of the 31 golfers can be constructed. Of these, 311 compared EE with other golfers, and 149 compared BG with another golfer. There are no comparisons of EE and BG. Only 4.5% of the 5,220 comparisons are ties. Application of (3) to the network of 4,929 paired comparisons without ties produces the finding that EE is estimated to be 1.15 logits more able than BG, and so EE is likely to be higher ranked than BG 3 times out of 4 were they to meet in a golf tournament under similar conditions.

This analysis is fast and easy to conceptualize, but has several drawbacks. First the ranking must be decomposed into paired comparisons. This may require some computer programming. A more fundamental problem is that paired comparisons within a ranking are not actually independent as (1) or (3) imply, but are required to be self-consistent.

## Rank Orders as Multiple Dependent Paired Comparisons

Consider the ranking of three golfers who played in the same three tournaments: Graeme McDowell, David Howell and Jean-Francois Remesy. They never tied. From the ranking of each tournament, we can construct three paired comparisons for these players. But they are not independent. From two of the comparisons built from a ranking we can often deduce the third comparison.

Table 1 shows the sample space for the independent paired comparisons of A, B and C (without ties). The probability of observing any particular ranking, e.g., A>B>C is

$$P(Ranking_{ABC}) = P_{ABC} / \{P_{ABC}\} \tag{5}$$

The likelihood of the set of *N* rankings of three objects is:

$$\Lambda = \prod_{r=1}^{N} P(r) / \{P_{ABC}\} \tag{6}$$

with log-likelihood, canceling out the common divisor:

$$\lambda = \sum_{r=1}^{N} \sum_{y=ABC} X_{yr} B_y - N \log \sum_{s=1}^{Sum} e^{\sum_{y=ABC} X_{ys} B_y} \tag{7}$$

Table 1

*Probabilities of Rankings of 3 Objects as Paired Comparisons*

| Self-consistent within ranking | Relative Probability | Self-inconsistent within ranking | Relative Probability |
|---|---|---|---|
| A>B B>C A>C | $P_{ABC} = P_{AB}P_{BC}P_{AC}$ | | |
| A>C C>B A>B | $P_{ACB} = P_{AC}P_{CB}P_{AB}$ | | |
| B>A A>C B>C | $P_{BAC} = P_{BA}P_{AC}P_{BC}$ | A>B B>C C>A | $P_{AB}P_{BC}P_{CA}$ |
| B>C C>A B>A | $P_{BCA} = P_{BC}P_{CA}P_{BA}$ | A>C C>B B>A | $P_{AC}P_{CB}P_{BA}$ |
| C>A A>B C>B | $P_{CAB} = P_{CA}P_{AB}P_{CB}$ | | |
| C>B B>A C>A | $P_{CBA} = P_{CB}P_{BA}P_{CA}$ | | |
| Only self-consistent: normalizer | Sum = $\{P_{ABC}\}$ | | |

where $y$ is A, B or C and $X_{yr}$ is 2 for the highest element in ranking $r$, 1 for the middle element and 0 for the lowest element. This yields the maximum-likelihood equation:

$$\frac{d\lambda}{dB_y} = \sum_{r=1}^{N} X_{yr} - N \sum_{s=1}^{Sum} X_{ys} P_s \tag{8}$$

where $P_s$ is the probability of observing ranking $s$, i.e.,

$$P_s = e^{\Sigma X_{ys} B_y} / \sum_{t=1}^{Sum} e^{\Sigma X_{yt} B_y} \tag{9}$$

Equation (8) yields the usual result that the maximum likelihood estimate corresponds to the point on the latent variable where the expected score equals the observed score. Then, for Newton-Raphson estimation, and standard error computation:

$$\frac{d^2\lambda}{dB_y{}^2} = N\left(\sum_{s=1}^{Sum} X_{ys} P_s\right)^2 - N \sum_{s=1}^{Sum} X_{ys}{}^2 P_s \tag{10}$$

For the three golfers, Graeme McDowell (GM), David Howell (DH) and Jean-Francois Remesy (JR), their observed rankings in the three tournaments were: JR>GM>DH, JR>DH>GM and DH>GM>JR. Regarding the paired comparisons as independent, their measures are JR = .47 (*SE* = .87), DH = .00 (.84) and GM = –.47 (.87), so that their range is .94 logits, and the statistical distance between JR and GM is $t = (.47 - -.47) / (.87^2+.87^2)^{½} = 0.76$.

Computing their pair-dependent measures, using Equations (8) and (10), yields JR = .35 (*SE* = .76), DH = .00 (.72) and GM = –.35 (.76). Thus modeling the dependency has made the measures more central (range = .70) and also made them more similar statistically, $t = (.35 - -.35) / (.76^2+.76^2)^{½} = 0.65$.

The expressions in Table 1 and equations (5)-(10) generalize to rankings of any length. For partial rankings, the multiplier "N" is replaced by a summation over all possible rankings of the elements in each partial ranking (see Linacre, 1994). As the rankings become longer, the proportion of self-consistent pairings to independent pairings reduces. For three objects (Table 1), the ratio is 6:8 = 1:1.3. For four objects, 24:64 = 1:2.7. For five objects, 120:1024 = 1:8.5. For 6 objects, 720:32,768 = 1:45.5. For 7 objects, 5,040 to 2,097,152 = 1:416.1. The number of terms in the normalizer (needed for computing expected scores) is the number of self-consistent pairings, so with three objects, this is 6, but with 13 objects, the number is 6,227,020,800, which is bigger than a standard computational "long integer". Thus the burden of computing self-consistent pairings becomes overwhelming as the rankings become longer.

The self-consistent-pairings approach was implemented in the computer program FRANK (Linacre, 1989). Linacre (1994) presents the analysis of rankings of 7 Baseball Announcer using the self-consistent-pairs approach to rankings. The range of measures is given there as 1.85 logits: 0.98 (.41) to –0.87 (.37). When the same dataset is analyzed using an independent–pairs approach (using the *Facets* program), the range of measures is 3.82 logits: 2.01 (.52) to –1.81 (.48). Figure 1 shows the relationship of the two sets of measures. It is seen to be close to linear.

## Independent Paired Comparisons with Ties

Returning to the golf tournaments, Nick Price (NP) played Colin Montgomerie (CM) in the WGC Match-Play Championship. What is the prediction? In the 47 stroke-play tournaments, NP ranked higher twice, and CM once, so NP's measure is estimated as log (2/1) = 0.7 logits better than CM's. But in a fourth tournament, they obtained the same ranking.

Applying the logic of Andrich (1978), there emerges a "rating scale" paired-comparison model which allows for ties:

$$\log (P_{nmj} / P_{nm(j-1)}) = B_n - B_m - F_j \tag{11}$$

where the categories are $j$ = 0,1, 2 for "ranked worse", "tied", "ranked better". $F_j$ is the Rasch-Andrich threshold, i.e., the point of equal probability on the latent variable, for categories $j-1$ and $j$. $F_0$ is set to zero or any convenient value. This model can be implemented directly in *Facets.*

Applying the paired-comparisons-with-ties model, the logit distance between $B_{NP}$ and $B_{CM}$ reduces from 0.7 logits to 0.34 logits. $F_1$ and $F_2$ are +.34 and –.34. Inferring back from the model parameter estimates,

$$\hat{P}_{nmj} = \frac{e^{j(\hat{B}_n - \hat{B}_m)) - \sum_{k=0}^{j} \hat{F}_k}}{\sum_{h=0}^{2} e^{h(\hat{B}_n - \hat{B}_m)) - \sum_{k=0}^{h} \hat{F}_k}} \qquad (12)$$

so that $P_{nm0} = e^0/(e^0+e^{.34-.34}+e^{2*.34}) = 0.25$, $P_{nm1} = e^{.34-.34}/(e^0+e^{.34-.34}+e^{2*.34}) = 0.25$, $P_{nm2} = e^{2*.34}/(e^0+e^{.34-.34}+e^{2*.34}) = 0.5$. This prediction matches the stroke-play data: NP is predicted to rank higher in 50% of their encounters, CM to rank higher in 25%, and for them to tie in 25%. In fact, in the WGC Match-Play Championship, they were tied at the end of the regulation 18 holes of play, but CM won on the 20th hole.

Regarding rankings as independent paired comparisons, this model can be applied to the performances across all 47 tournaments of the 30 players who played in the Mercedes Tournament. This yields 5,071 paired comparisons. Direct analysis of these data with the model in (11) by *Facets* program yields a measure range of 1.42 logits: 0.81 (.07) to –0.61 (.12). Formatting the same data as a rectangular data set (paired performances as rows, golfers as columns) and using JMLE estimation, produces a measure range of 2.88: 1.64 (.07) to –1.24 (.12). This supports the same "rectangular" JMLE estimation bias correction as applied to independent pairs without ties.

## Dependent Paired Comparisons with Ties

When ties are allowed, there are no longer 8 possible combinations for 3 objects (as in Table 1), but 27, and not merely 2 of them inconsistent, but 14. Consequently, in general, the computational load with ties is greater for dependent pairings and the estimation bias less predictable for independent pairings.

There were 12 tournaments in which Stuart Appleby (SA), Jonathan Kaye (JK) and Kirk Triplet (KT) all played. These produce 36 paired comparisons of the 3 players. 6 of these comparisons are ties. Scoring these comparisons, 2 when ranked higher, 1 when tied, and 0 when ranked lower, then SA scored 36, KT 24, and JK 12. When analyzed as independent paired comparisons using (9) in *Facets,* these produce a measure range of 0.96 logits: 0.48 (.27) to –0.48 (.27). When analyzed as dependent pairs, the measure range is more central: 0.67 logits: 0.34 (.23) to –0.34 (.23).

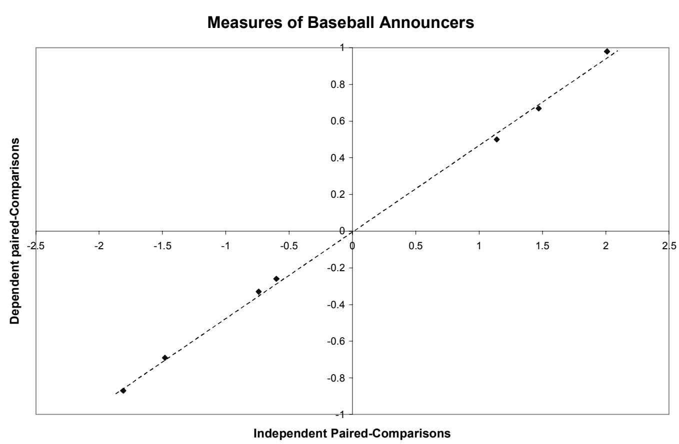


*Figure 1*. Paired-Comparison Measures.

## Long Rankings as Rating Scales

The computational load for dependent paired comparisons grows exponentially as the number of objects in the rankings increases. For independent paired comparisons, the proportion of inconsistent pairings increases with increasing length of rankings. An attractive alternative approach is to treat rankings as rating scales.

Imagine that each golf tournament is a survey item, and that each golfer is being assigned to a rating scale category according to his performance. Then, if there are no ties, each rater is assigned to his own category, and the category number matches the golfer's ranking in the tournament. If there are ties, then two or more golfers are assigned to the same category. Applying this approach to the threesome of SA, JK and KT, and ranking them 1, 2 or 3, they produce the measure range 2.32 logits: –1.16 (.48) to 1.16 (.48). The increase in measure range relative to paired comparisons can be attributed in large part to the rating-scale approach being "rectangular", i.e., modeling the golfers playing against the tournament, rather than against each other.

It might be thought that the three different measure ranges for the dependent-pairs, independent-pairs, and rating-scale approaches would lead to different inferences, and they do in part. The parameter estimates for all three approaches recover the observed marginal scores. However, they do not report the same quality-control fit statistics, nor predict the same relative success in the future. A wider measure range for an approach implies that the approach perceives the data to be less stochastic, i.e., to have more of a Guttman pattern and be more predictable. If, according to one approach, the data were perfectly predictable, then the measure range reported by that approach would be infinite. Thus, the wider the reported measure range, the more unexpected statistically are the irregular results, such as Stuart Appleby's poor performance in the U.S. Open, and the more expected are the predicted results. This is because, for all three of these approaches, the standardized residuals are modeled to have a mean of zero and standard deviation of 1.

## Measures for 356 Golfers across 47 Tournaments Paired Comparisons

Of the 357 golfers who played in two or more tournaments, one player, Arnold Palmer, scored worse in both his tournaments than any of the other 356 golfers. Thus his measure is not estimable in the same way as the other golfers, and he is dropped from this analysis. The rankings of the 356 multi-tournament golfers for the 47 tournaments produce 391,374 paired comparisons. Of these, 16,639 are ties. Since, on average, 133 players participated in each Tournament, it is impractical to compute the measures based on dependent paired comparisons.

Treating the paired comparisons as independent, the measure range was 3.87 logits from Tiger Woods (TW) at 1.57 logits (.04) and Vijay Singh (VS) at 1.39 logits (.03) down to Brad Hauer at (BH) –2.30 logits (.32). According to the conventional Rasch-model parameter-level fit statistics, Mike Baker Jr. (MB) was the most consistent in his performance across tournaments. He played in 2 tournaments, and was ranked 145 and 147. Tiger Woods was almost as consistent across his 18 tournaments. Thongchai Jaidee was least consistent. He played in 5 tournaments and was ranked 32, 41, 80, 139 and 147. The golfers played most consistently in line with their overall performance at the Western Open. They played least consistently at the WGC-NEC Invitational, where top players, Ernie Els and Padraig Harrington, ranked unexpectedly low at 65th and 73rd out of the 76 players participating. Appendix A shows a *Facets* control file for this analysis.

Analyzing the paired comparisons as a conventional rectangular dataset with golfers as items and pairings as rows, the performance range is 7.75 logits, twice the direct comparison range of 3.87 logits, as expected. Again, Thongchai Jaidee is reported as least consistent and Mike Baker Jr. as most consistent. Appendix B shows a *Winsteps* control file for this analysis.

## Measures for 356 Golfers across 47 Tournaments Rankings

Another approach is to model each tournament as a performance item on which the golfer's rank is the rating. Analytically, the 47 tournaments become 47 items. The worst rank at any tournament by one of multi-tournament players was 174 at the Pebble Beach National Pro-Am, but this tournament included one-tournament-only players and also tied ranks. When this tournament is rescored so that the ranking includes only multi-tournament players and is strictly ordinal (with no unobserved intermediate rankings), the worst rank becomes 37.

Each of the other 46 tournaments is rescored this same way. The "International" tournament has the most different levels of performance, 43. Accordingly, each tournament is modeled to define its own rating scale, i.e., to accord with a "partial credit" model (Masters, 1982).

Appendix C shows the observed rankings formulated for analysis by *Winsteps.* The analysis of these data produces results which match earlier findings. Tiger Woods is measured highest at .40 logits (.07) and Brad Hauer lowest at –1.53 logits (1.05). This measure range is 1.93 logits, about half of the 3.87 logit range of the independent paired comparisons, but this is somewhat misleading. Figure 2 shows that there is a sigmoid relationship between the two sets of measures. The paired comparisons show a large ability range between Brad Hauer (BH) and Mike Baker Jr. (MB). They played in only two tournaments each, but not in the same ones. In one tournament BH was equal last, otherwise they were close to last. In this instance, their small separation on the rank-order metric is more reasonable than is their large separation (almost half the ability range) on the paired-comparison metric. It appears that the paired-comparison measures are exaggerated at the extremes of the ability range.

Mike Baker, Jr., is again reported as a most consistent golfer. The most inconsistent performer according to this approach is Tetsuji Hiratsuka. In his four tournaments he ranked 36, 129, 131, and 133. In the paired-comparison approaches the golfer reported as most inconsistent was Thongchai Jaidee, ranked 32, 41, 80, 139 and 147. It is seen that the paired-comparison approach is more sensitive to misfitting performance profiles, but the rank-order approach identifies aberrant individual performances as misfitting. From the tournament organizer's viewpoint, it seems prudent to treat Thongchai Jaidee is a golfer who performs roughly at an "80" level and to treat Tetsuji Hiratsuka as a golfer who generally performs at a "130" level, but has had one excellent performance, which he is unlikely to repeat. But, as things stand and disregarding misfit, Tetsuji Hiratsuka is reported to be a better player than Thongchai Jaidee.

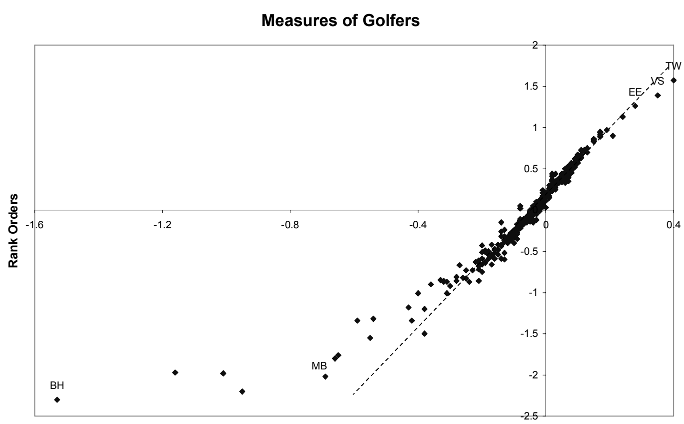


*Figure 2.* Independent Paired-Comparisons vs. Rank Orders

## Conclusion

Four approaches to the analysis of rank-order data have been presented here. It has been demonstrated that the two independent-paired-comparison approaches, the direct and the rectangular, are equivalent with the application of a correction factor.

The more rigorous dependent-paired-comparison approach is appealing, but is computationally impractical for long rankings. Figure 1 indicates that dependent-paired-comparison measures have a close-to-linear relationship with independent-paired-comparison measures.

The rank-order approach is implemented as a rectangular dataset and so has an inherent estimation bias. Nevertheless, Figure 2 indicates that the measures it estimates are more plausible for long rankings than those produced by the independent-paired-comparison method, particularly in the tails. The misfit identified by this approach also appears to have more immediate practical relevance than that identified by paired comparisons.

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## Appendix A
## Facets control and data file for independent paired comparisons

```
title = "47 Golf Tournaments"
facets= 3          ; data set up with 3 facet-element locations
entered=1,1,2      ;in data, facet one (golfer), facet one (golfer), facet two (tournament)
model=?,-?,?,R2,0.5   ; which means:
        ; any element in the first facet position (golfer) can be opposed by any element in the second
        facet position (golfer) in the context of the third facet position (tournament) with rating on
        a rating scale with highest category 2, and each observation weighted 0.5.
noncentered=0  ; all facets are centered
positive=0     ; all facets are negative: low score = high measure
arrange=m,f    ; report measures in measure order and fit order
labels=
1,golfers      ; Facet one is 356 golfers
 1, Benoit Beisser
 2, Bill Britton
 ......
 356, Tom Carter
*
2, Tournaments, A      ; Facet two is 47 Tournaments
 1, Mercedes Championships,0      ; Anchored at 0, used for fit reporting only.
 2, Chrysler Classic of Tucson,0
 .....
 47, Michelin Championship at Las Vegas,0
*
data=   ; all data points entered twice, weighted 0.5;
; first golfer, second golfer, tournament, comparison (0=ranked lower, 1=tied, 2=ranked higher)
 94, 103, 1,2     ; golfer 94 paired with golfer 103 at tournament 1 was ranked higher
 103, 94, 1,0     ; golfer 103 paired with golfer 94 at tournament 1 was ranked lower
 94, 138, 1,2
 138, 94, 1,0
 ......           ; 782,748 data lines
 353, 355, 45,2
 355, 353, 45,0
 354, 355, 45,0
 355, 354, 45,2
```

## Appendix B

### Winsteps control and data file for independent paired comparisons

```
TITLE = "Golf 2004 Paired-comparison"
ni =  356     ; Players are the columns
xwide = 1     ; each observation one column wide
item1 = 1     ; first player in column 1
name1 = 357   ; tournament number and name starts in column 357
CODES= 012    ; valid codes
CLFILE=*      ; descriptions of the codes
0 = ranked lower
1 = tied
2 = ranks higher
*
Paired = Yes  ; automatic estimation bias correction
&end
  1 Benoit Beisser
  2 Bill Britton
 ......
356 Tom Carter
END LABELS
; only two observations per line: one for each golfer in the pairing
02                               01 Mercedes Championships
0 2                              01 Mercedes Championships
.......
(382,182 data lines)
```

## Appendix C

Winsteps control and data file for ranked data

```
TITLE = "Golf 2004 Ranked data"
ni = 47          ; 47 tournaments
xwide = 3        ; 3 columns per observation (1-254)
item1 = 1        ; first tournament in columns 1-3
name1 = 142      ; golfer identification
Groups = 0       ; each tournament has its own rating scale (partial credit)
Stkeep = No      ; intermediate unobserved categories dropped out
                        ; these are due to ties and one-tournament golfers
; next are all ranks from 1 to 254. Highest number observed was 174
CODES="  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23+
 + 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47+
 + 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71+
 + 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95+
 + 96 97 98 99100101102103104105106107108109110111112113114115116117118119+
 +120121122123124125126127128129130131132133134135136137138139140141142143+
 +144145146147148149150151152153154155156157158159160161162163164165166167+
 +168169170171172173174175176177178179180181182183184185186187188189190191+
 +192193194195196197198199200201202203204205206207208209210211212213214215+
 +216217218219220221222223224225226227228229230231232233234235236237238239+
 +240241242243244245246247248249250251252253254"
&end
 1 Mercedes Championships
 2 Chrysler Classic of Tucson
....             ; 47 tournaments
47 Michelin Championship at Las Vegas
END LABELS
; tournaments
; 1  2  3  4  5  6  7  8  9 ...47 (141 data columns, then the golfers' names)
 21 34  9108116          34 .....J.L. Lewis
 21       71 25       59    .....Fred Couples
 24    41  5  4          1  .....Mike Weir
 25  3   101 86 21    15  3 .....Rory Sabbatini
....
(356 data lines)
```